Section 9.1

Quadratic Function

A quadratic function of the variable x is a function that can be written in the form

$$f(x) = ax^2 + bx + c \quad \text{or} \quad y = ax^2 + bx + c.$$

Features of a Parabola

The graph of $f(x) = ax^2 + bx + c$ with $a \neq 0$ is a parabola. If $a > 0$ the parabola opens upward (concave up) and if $a < 0$ it opens downward (concave down).

Vertex

The vertex is the turning point of the parabola. Its coordinates are given by:

$$(x, y) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

x-intercepts

These occur when $f(x) = 0$; that is when $ax^2 + bx + c = 0$. Use the quadratic formula to find the x-intercepts. The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If the discriminant $b^2 - 4ac$ is positive, there are two x-intercepts. If it is zero, there is a single x-intercept, and the vertex touches the x-axis. If it is negative, there are no x-intercepts, the parabola doesn't touch or intersect the x-axis.

y-intercept

This occurs when $x = 0$, so $y = a(0)^2 + b(0) + c = c$. So the y-intercept is at the point $(0, c)$.

Symmetry

The parabola is symmetric with respect to the vertical line through the vertex, which is the line $x = -\frac{b}{2a}$.

Problem 1. Sketch the graphs of the quadratic functions, indicating the coordinates of the vertex, the y-intercept, and the x-intercepts (if any).

a) $f(x) = x^2 - 4$

b) $f(x) = -x^2 + 4x - 4.$

c) $f(x) = -x^2 + x - 1$

d) $f(x) = x^2 + 1$

Problem 2. The average weight of a sedan could be approximated by

$$W = 6t^2 - 240t + 4800 \qquad (5 \leq t \leq 27)$$

where t is its year of manufacture ($t = 0$ represents 1970) and W is the average weight of a sedan in pounds. Sketch the graph of W as a function of t. According to the model, in what year were sedans the lightest? What was their average weight in that year?

Problem 3. The market research department of the Better Baby Buggy Co. predicts that the demand equation for its buggies is given by $q = -0.5p + 140$, where q is the number of buggies it can sell in a month if the price is $\$p$ per buggy. At what price should it sell the buggies to get the largest revenue? What is the largest monthly revenue?

Problem 4. The latest demand equation for your gaming website, www.mudbeast.net, is given by

$$q = -400x + 1{,}200$$

where q is the number of users who log on per month and x is the log-on fee you charge. Your Internet provider bills you as follows:

Site maintenance fee: $20 per month

High-volume access fee: 50¢ per log-on

Find the monthly cost as a function of the log-on fee x. Then, find the monthly profit as a function of x and determine the log-on fee you should charge to obtain the largest possible monthly profit. What is the largest possible monthly profit?

Problem 5. Find the quadratic regression curve through the given points. $\{(1,2),(3,5),(4,3),(5,1)\}$.

Problem 6. Phone companies have been scrambling to install fiber-optic cable in order to compete with television cable companies. The following table shows the number of fiber-optic cable connections to homes in the U.S. from 2000 to 2004 ($t = 0$ represents 2000):

Year t	0	1	2	3	4
Connections (Thousands)	0	10	25	65	150

a) Find a quadratic regression model for these data. Graph the model together with the data.

b) Assuming the trend had continued, estimate the number of connections in 2005 to the nearest 1000 homes.

Homework for this section: Read the section and watch the videos/tutorials. Then do these problems in preparation for the quiz:#4, 12, 16 (on Excel), 22, 26, 32, 34, 40 (on Excel)